# 4.8 - Analyzing Graphs of Polynomial Functions



Zeros, factors, solutions, and x-intercepts are closely related concepts.

## Zeros, Factors, Solutions, and Intercepts

Let $f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$ be a polynomial function. The following statements are equivalent.

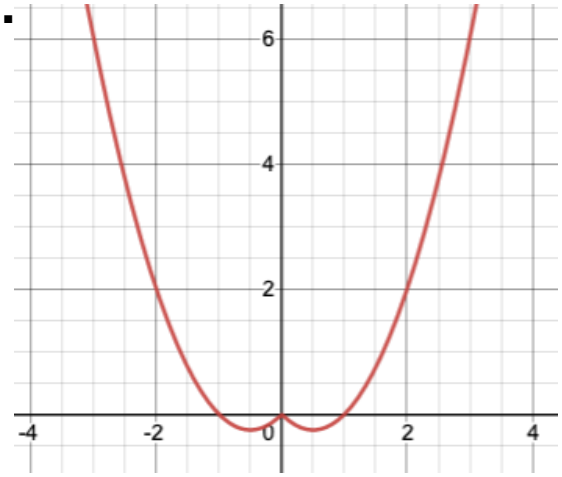
| | |
|---|---|
| **Zero:** | $k$ is a zero of the polynomial function $f$. |
| **Factor:** | $x - k$ is a factor of the polynomial $f(x)$. |
| **Solution:** | $k$ is a solution (or root) of the polynomial equation $f(x) = 0$. |
| ***x*-Intercept:** | If $k$ is a real number, then $k$ is an $x$-intercept of the graph of the polynomial function $f$. The graph of $f$ passes through $(k, 0)$. |

# 4.8 - Analyzing Graphs of Polynomial Functions



## Three Symmetries

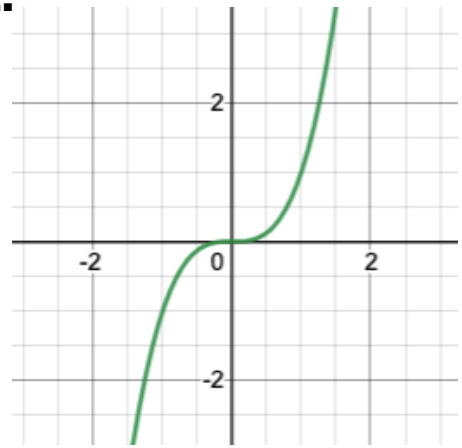
**1.**




**y-axis**

**even**

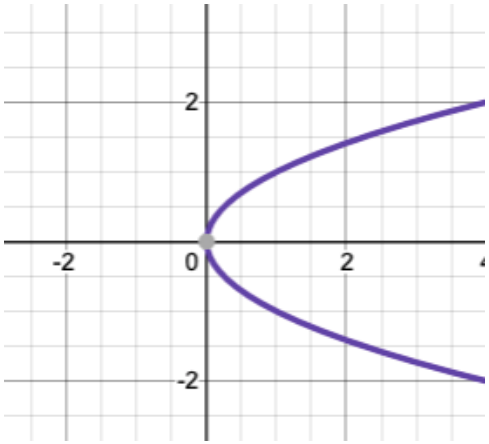
**2.**




**origin**

**odd**

**3.**




**x-axis**

**not function**

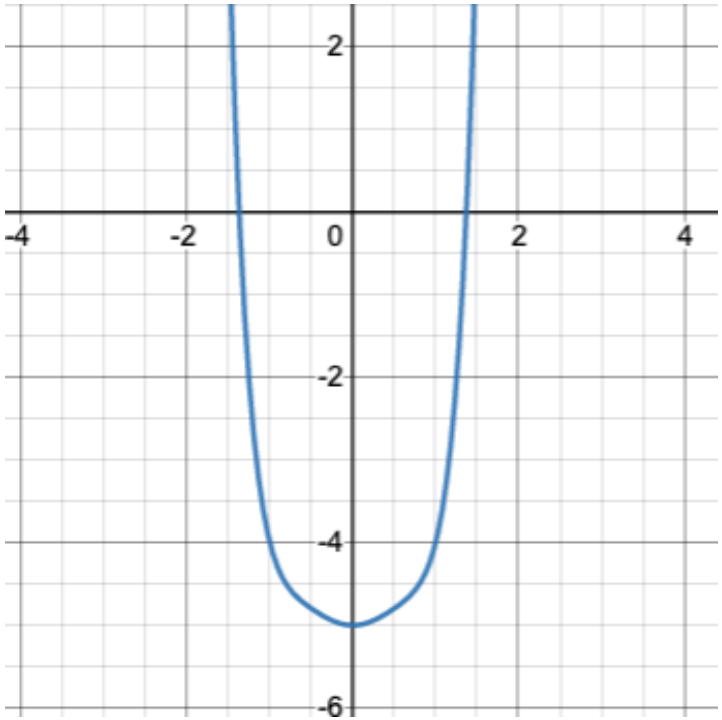
# 4.8 - Analyzing Graphs of Polynomial Functions



## Even Functions

$$f(-x) = f(x)$$

$$f(x) = x^6 - x^4 + x^2 - 5$$



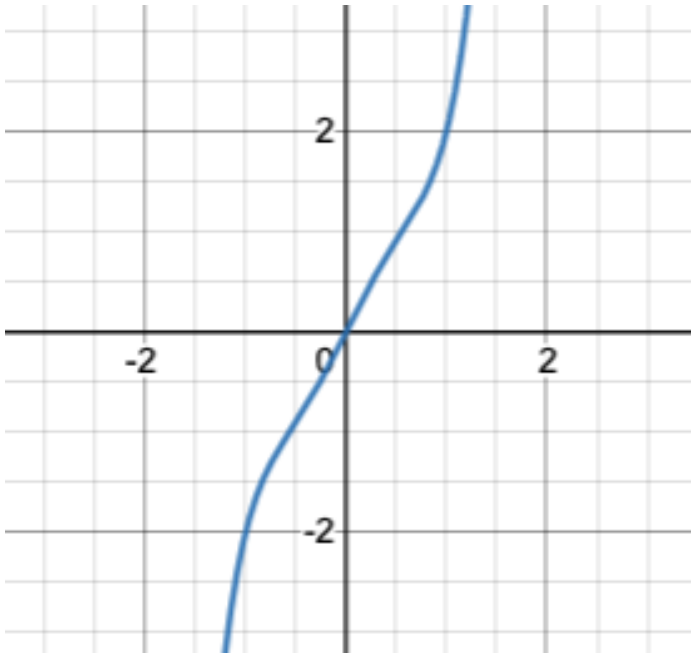
# 4.8 - Analyzing Graphs of Polynomial Functions



## Odd Functions

$$f(-x) = -f(x)$$

$$f(x) = x^5 - x^3 + 2x$$



# 4.8 - Analyzing Graphs of Polynomial Functions



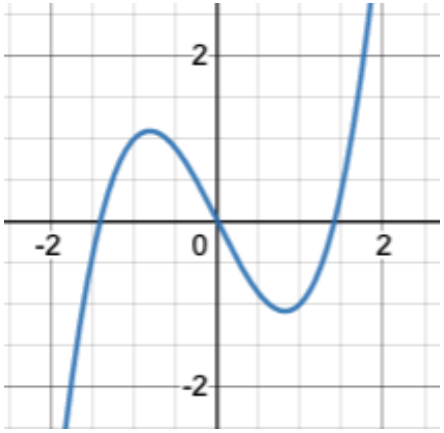
## Even or Odd?

1. $y = x^3 - 2x$

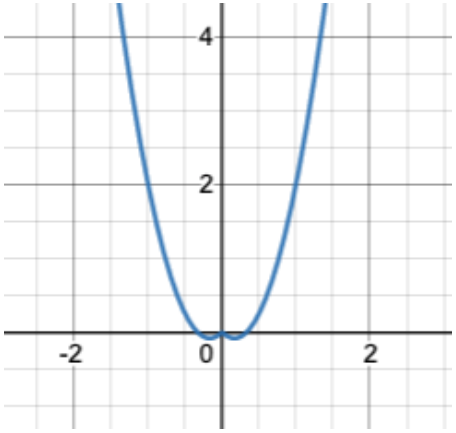
2. $y = -|x| + 3x^2$

3. $y = x^2 + \frac{1}{x^4}$

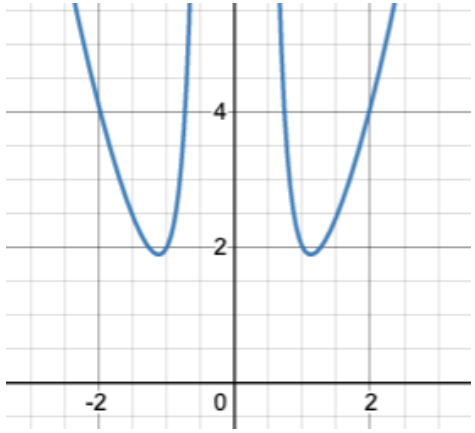
**Odd**




**Even**




**Even**



# 4.4 - Factoring Polynomials



## Practice: Factoring

1. $3x^6 - 2x^3 - 1$

$(3x^3 + 1)(x - 1)(x^2 + x + 1)$

2. $\frac{2}{y^4} - \frac{3}{y^2} + 1$

$(\frac{2}{y^2} - 1)(\frac{1}{y} - 1)(\frac{1}{y} + 1)$

# 4.2 - Adding, Subtracting, and Multiplying Polynomials



## Practice: Binomial Theorem

1. $(x+2y)^4$

$x^4+8x^3y+24x^2y^2+32xy^3+16y^4$

2. $(2x-3y)^{14}$

Find 4th term

$-20{,}127{,}744x^{11}y^3$

# 4.5 - Solving Polynomial Equations



## Practice: Rational Root Theorem

List all possible rational roots, then solve.

$$2x^4 + 9x^3 + 15x^2 + 6x - 8 = 0$$

$$\frac{p}{q} = \pm\left\{1,2,4,8,\frac{1}{2}\right\}$$

$$Roots = \left\{-2,\ \frac{1}{2},\ \frac{-3 \pm i\sqrt{7}}{2}\right\}$$

# 4.3 - Dividing Polynomials



## Practice: Synthetic Division

$$(2x^4 + 9x^3 + 15x^2 + 6x - 8)/(x - 3)$$

$$2x^3 + 15x^2 + 60x + 186 + \frac{550}{x - 3}$$

# 4.3 - Dividing Polynomials



## Practice: Long Division

$$(2x^4 + 9x^3 + 15x^2 + 6x - 8)/(x^2 + x - 3)$$

$$2x^2 + 7x + 14 + \frac{13x + 34}{x^2 + x - 3}$$